

The Ticker-Tape Interpretation of Quantum Mechanics

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In recent years there has been a move away from the Copenhagen Interpretation towards alternative interpretations of Quantum Mechanics. There has also been an acknowledgement that there is no definitive version of the Copenhagen Interpretation because the originators, Bohr, Heisenberg et. al., did not agree over all aspects of the interpretation. This paper revisits the philosophical approach taken by Bohr and Heisenberg. The result is a new interpretation, named the Ticker-Tape Interpretation, which is closely related to Copenhagen. The interpretation leads to some conjectures.

Author’s Note

The author has rather playfully borrowed the title of some of Einstein’s famous “Principles”. Even though the principles in this paper have the same names as those found in Special or General Relativity, *they have nothing to do with any specifics of Special or General Relativity*; the names were chosen however because, at some level, the both versions derive from an even more general principle.

1 Philosophical Underpinnings

This paper adopts a world view that is essentially Kantian. The Kantian world view consists of an external world that is perceived through the sensors of an agent; the agent builds mental models of the external world but cannot know its true nature.

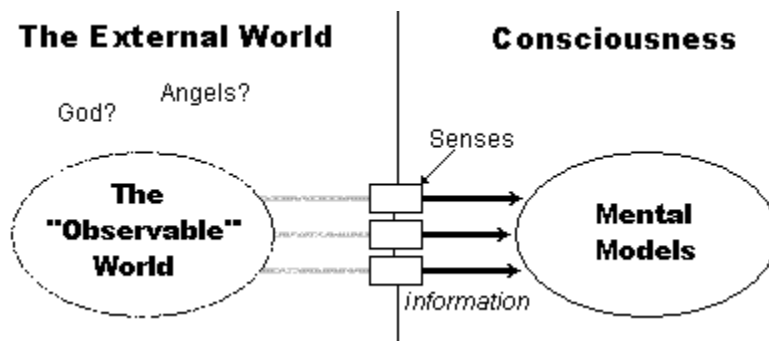


Figure 1.
Kantian World View (circa 1785).

Copenhagen clarifies aspects of the Kantian world view. (It has been argued that the Copenhagen Interpretation is Positivist, however subtle arguments over philosophical classifications are beyond the scope of this paper) Quantum Mechanics replaces the vague idea of sensory input with that of measurement and precise mathematical description. The idea of a conscious agent is replaced by intelligence, which need not be human (although some commentators would dispute this). It could, for example, be a robot.

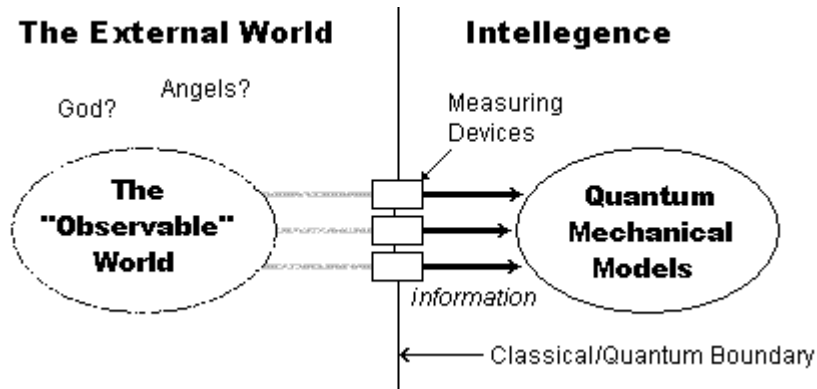


Figure 2.
Copenhagen World View (circa 1928).

The Copenhagen Interpretation acknowledges that our mental models may be incomplete; it may be necessary to apply a wave model in some situations, and apply a particle model in another (Complementary Principle). Realism, on the other hand, is rooted in the belief that the world is as it appears, and seeks to preserve macroscopic models, possibly beyond their domain of applicability.

2 Measurement

Measurements are regarded as “elements of reality”, although no definition of what this means is given. The observer is logically separate from the measurement itself. A measurement discontinuously changes the probability distribution associated with a “physically significant” random variable.

Definition: A measuring device that produces a single representative number as its output is called a “basic” measuring device, and the measurements it produces are called “basic” measurements. The notation $\mathbf{X} = x$ is sometimes used to mean a basic measurement of the physical quantity denoted by \mathbf{X} has been made and reported as x .

Definition: A history is a sequence of measurements and denoted $\mathbf{H} = (\mathbf{m}_0, \mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_n)$ where \mathbf{m}_i are measurements. If all measurements are “basic”, the history will look like $(\mathbf{X}_0 = x_0, \mathbf{X}_1 = x_1, \mathbf{X}_2 = x_2, \dots, \mathbf{X}_n = x_n)$.

3 Mach Devices

Bohr expressed the opinion that measuring devices are essentially classical, and that a description of reality required a-prior understanding of classical quantities such as position and momentum ^[1]. There are difficulties with this approach. For example, the approach requires a knowledge of classical mechanics before Quantum Mechanical measurements can be understood, yet Quantum Mechanics is presumed to be more fundamental than classical physics. It is also not easy to apply Bohr's vision of essentially classical measuring devices to abstract concepts such as QCD colour.

Rather than follow Bohr or Kant, we take a slightly different view of measurement.

Definition: A measuring device is a Mach device *with respect to an observer* if the following applies:

- (i) The device produces a stream of measured values (numbers) that are recorded on a ticker-tape (or equivalent) accessible to the observer. Measurements are recorded in the order they are made.
- (iii) The device's internal structure is unknown. It is a "black box". There is no *a-prior* information about what the numbers it produces mean.
- (iv) The observer does not have access to a clock.



Figure 4
The output of a Mach device viewed as a "ticker tape".

A Mach device is the most primitive measurement device possible: it only produces basic measurements. Why? The answer goes to the heart of what measurement is: measurements can be put "in order" from the smallest to the largest. I.e. If $\mathbf{M} = \{\text{set of possible measured values}\}$, then there is a strict total ordering $<$ of \mathbf{M} , which creates an order preserving isomorphism from \mathbf{M} to the some subset of Real numbers.

Mach devices cannot return probability distributions since probability distributions implicitly refer to an underlying random variable and so contains information that, by definition, a Mach device cannot provide.

Even though the result of a measurement by a Mach device is a single number, the interpretation of the resulting numeric label is not simple.

Suppose \mathbf{X} and \mathbf{Y} are Mach devices relative to observer A except that it is known that \mathbf{X} and \mathbf{Y} measure the same quantity. Suppose further that the same devices are not Mach devices with respect to a second observer B who also has access to third measuring device \mathbf{Z} with a far higher resolution. The measurements of \mathbf{X} and \mathbf{Y} with respect to observer B may correspond to the selection of probability distributions like those below (figure 5).

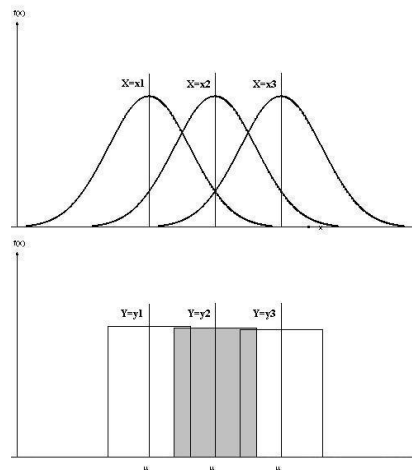


Figure 5.
Differing distributions for numeric "labels".

It is also possible that the output of one of the devices \mathbf{X} could represent discrete values such as electron spin (numeric label = measured value), or that no “high-resolution” third device \mathbf{Z} exists.

A measurement can be thought of as a pair (\mathbf{X}, φ) where \mathbf{X} represents the device used to measure the physical quantity and φ is a probability density function.

The issue of interpretation is related to the question “What does a measuring device measure?”

4 The Equivalence Principle

There is no transition from the Quantum world to the Classical world. The difference between measurement in the Classical world and measurement in the Quantum world is a matter of interpretation.

Quantum Mechanics is a mental model; the waveform and any measurement operator \mathbf{A} are mental constructions, built on top of the information gathered from “raw” measurements. Classical mechanics is also a mental model (since we know it is not “true”, it cannot be otherwise); typically Classical Mechanics deals with $\langle \mathbf{A} \rangle$ and regards the difference from $\langle \mathbf{A} \rangle$ to be “error”.

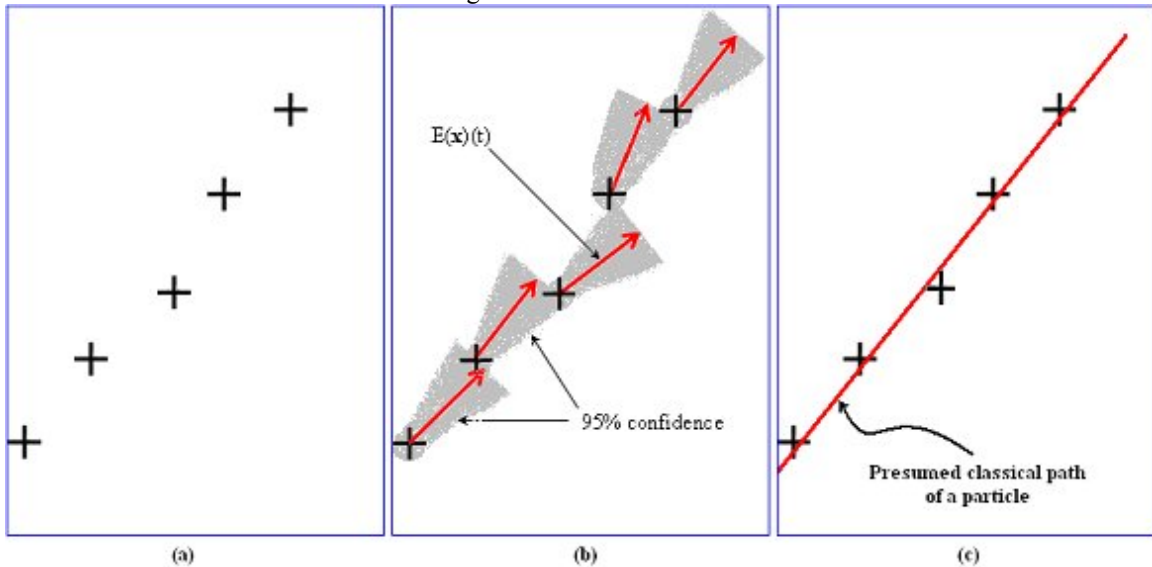


Figure 6
Measurements and Mental Models.

In the diagram above, the left pane shows the “raw” position measurements of a particle moving diagonally from left to right is shown. The central pane *adds* the intellectual machinery of Quantum Mechanics; the *calculated* 95% confidence intervals are shown in gray. The right pane *adds* the presumed classical path.

In Quantum Mechanics, the uncertainty in the position of a particle created by Heisenberg Uncertainty Principle is viewed as intrinsic to the system; the measurement themselves are taken at face value (*The Principle of Exact Measurement*). In Classical Mechanics, the “error” (uncertainty) in the position of the particle has a multitude of sources, typically related to the construction of the measuring devices and lack of knowledge of initial conditions, but there is a presumption that if these influences could be eradicated, exact measurements would be possible.

5 Principle of Relativity (Mach's Principle)

The output from a single Mach device is meaningless.

This is a generalisation of Mach's Principle. Mach's statement of the Principle of Relativity famously influenced Einstein but the principle itself dates back at least to Galileo. Ernst Mach argued that it would be meaningless to talk about the motion of a single particle in an empty Universe. All motion is relative. In fact, all measurement is relative. If there is no context, a measurement stream becomes a meaningless stream of numbers.

6 Mach Banishes Determinism

Suppose $X = f(t)$ is a classical quantity, and \mathbf{X} is a measuring device that faithfully returns X . If \mathbf{X} is a Mach device with respect to an observer, it is not possible for that observer to determine what the device measures from the measurement history. Why? There is no way to calibrate the device.

Suppose we construct a second device \mathbf{Y} whose output is related to the first by

$$\mathbf{Y} = \zeta(\mathbf{X}(t))$$

The second device is sealed, mixed up with the first and given to the naive observer so they both become Mach devices. Which measures the "fundamental" quantity? \mathbf{X} and \mathbf{Y} ? \mathbf{X} ? \mathbf{Y} ? In fact, we can build the device \mathbf{Y} so it returns any measurement profile we like.

*If a measuring device qualifies as a Mach device **except** that it is known that it measures a quantity X where $X = f(t)$ for some function f , then the output from that device is meaningless.*

7 State

Definition: A system state is *any* representation Ψ such that $P(\mathbf{x} | \Psi) = P(\mathbf{x} | \mathbf{H})$ for all \mathbf{x} where \mathbf{H} is the known history of the system.

The minimal sub-history Ψ of \mathbf{H} such that $P(\mathbf{x} | \Psi) = P(\mathbf{x} | \mathbf{H})$ for all \mathbf{x} is one possible system state representation.

The test of a good theory is whether it can make accurate predictions. In the case of Quantum Mechanics, the obvious question is: How much history is necessary before an observer can make accurate predictions? The answer cannot be that the observer must know the entire history of a system since the beginning of time since that information will never be available. So what are the alternatives? One possibility is to only count the last N measurements for some N , perhaps giving the more recent measurements more "weight". But how do we choose N ? How do we "weight" different measurements? The issue of numeric stability also raises its head. One class of measuring devices however provides us with an easy win and provides us with an obvious starting point for a definition of state..

Definition: \mathbf{A} and \mathbf{B} are conjugate devices if a measurement of \mathbf{A} "wipes out" the measurement of \mathbf{B} ., and a measurement of \mathbf{B} "wipes out" the measurement of \mathbf{A} . More precisely

$$\begin{aligned} & P(\mathbf{x} | (\mathbf{A}=a, \mathbf{B}=b)) = P(\mathbf{x} | \mathbf{B}=b) \text{ for all } \mathbf{x}, a, b \\ \text{and} \\ & P(\mathbf{x} | (\mathbf{B}=b, \mathbf{A}=a)) = P(\mathbf{x} | \mathbf{A}=a) \text{ for all } \mathbf{x}, b, a \end{aligned}$$

where the probabilities $P(X|Y)$ are derived via Bayesian analysis of a large number of previous measurements.

Corollary: The last measurement provided by a set of conjugate devices is a representation of the state of the system described by the devices.

8 Repeatability

A stream of numbers produced by a device does not automatically qualify as measurements. Measurements must be “repeatable”. The reason we make measurements is the confidence that if we repeated the measurement, there is a “high” probability that we will get the same result or a result that is “acceptable” close.

Mach devices do not come equipped with a clock so there is no sense of time; it is not possible to distinguish between a “slow” (read “acceptable”) drift from the origin value and “jump” to a new value.

Definition: A measurement device produces repeatable results if $P(\Psi|\Psi) = 1$ for any state Ψ .

9 The Calibration Problem – A solution

How is it possible to determine what the output of a Mach device represents? If the stream of measurement is deterministic, then the output from the device is meaningless. It seems that a measurement stream can only be “understood” if it contains a “random component”. But how is it possible to make sense of such a randomised stream?

Fortunately early 20th Century scientists have done most of the work for us.

We assert that it is possible to assign meaning to a measurement stream provided that

- (i) The measuring devices associated with the measurement stream are conjugate.
- (ii) The probability of transition from the state Ψ to state Φ is the same as the probability of transition from state Φ to state Ψ .

$$P(\Phi|\Psi) = P(\Psi|\Phi) \text{ for any states } \Psi, \Phi$$

A physical system, measurement device and observers that satisfy the condition above is called a quantum system.

10 The Formalism of Quantum Mechanics

Initially we consider only Quantum systems with stable state transition probabilities. In that case, the system is described by a state transition matrix and the last known measurement.

For example, a system with measuring devices **A** and **B** with binary values \uparrow, \downarrow can be described by the state transition matrix below, together with the last known measurement

State (history) (From)	Probability (To)			
	(A = ↑)	(A = ↓)	(B = ↑)	(B = ↓)
$\Psi_{A,\uparrow}$	1	0	0.5	0.5
$\Psi_{A,\downarrow}$	0	1	0.5	0.5
$\Psi_{B,\uparrow}$	0.5	0.5	1	0
$\Psi_{B,\downarrow}$	0.5	0.5	0	1

Figure 7
State Transition Matrix

It is a well known result that such a state transition diagram can be rewritten as the product of state vectors (TODO – get reference).

10.1 Algebraic Extensions

10.1.1 Principle of Relativity (No Preferred Frame of Reference)

If Ψ is a representation of state, then the rule for the calculation of $P((X,\varphi)|\Psi)$ should be the same as $P((Y,\chi)|\Psi)$ irrespective of the state Ψ , or devices X or Y .

Author's Note

Here the concept of “frame of reference” is used loosely to mean “set of measuring devices”.

The Principle of Relativity (No Preferred Frame of reference) is used to justify the application of the same calculation rules no matter which measuring devices are used. It is natural then to use algebraic extensions of the state space (Real numbers \rightarrow Complex numbers and beyond)

Suppose we have two conjugate Mach devices, denoted A and B . Each device can return one of two values, designated \uparrow and \downarrow (borrowed from spin, the archetypical quantum property). As before, the system is described by the table below.

State (history) (From)	Probability (To)			
	(A = ↑)	(A = ↓)	(B = ↑)	(B = ↓)
$\Psi_{A,\uparrow}$	1	0	0.5	0.5
$\Psi_{A,\downarrow}$	0	1	0.5	0.5
$\Psi_{B,\uparrow}$	0.5	0.5	1	0
$\Psi_{B,\downarrow}$	0.5	0.5	0	1

Figure 7
Transition Probabilities

If A and B are conjugate devices, A “wipes out” any knowledge of B , so the state space is spanned by $\{\Psi_{A,\uparrow}, \Psi_{A,\downarrow}\}$. It follows that

$$\begin{aligned} \Psi_{B,\uparrow} &= a.\Psi_{A,\uparrow} + b.\Psi_{A,\downarrow} && \text{for some } a,b \\ \Psi_{B,\downarrow} &= c.\Psi_{A,\uparrow} + d.\Psi_{A,\downarrow} && \text{for some } c,d \end{aligned}$$

Solving yields (non-unique):

$$\Psi_{A,\uparrow} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \Psi_{A,\downarrow} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \Psi_{B,\uparrow} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \Psi_{B,\downarrow} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

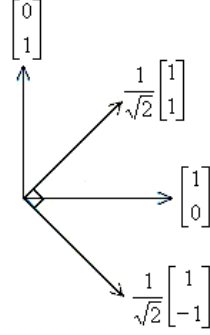


Figure 8
Geometric Interpretation of state space.

$$P(\Psi|\Phi) = |\langle \Psi|\Phi \rangle|^2$$

If a 3rd 2-valued conjugate device **C** is introduced, the relevant equations admit no real-valued solutions. The Principle of Relativity (no preferred frame of reference) however suggests that all devices and states should be treated equally; a particular choice of basis should not preclude a solution. Solutions do exist if the “co-ordinate” domain is expanded to included complex numbers (a standard practice in mathematics over the centuries). The resulting solution looks like:

$$\Psi_{C,\uparrow} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \Psi_{C,\downarrow} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

The measurement operators are found to be

$$\mathbf{L}_x = \hbar/2 \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{L}_y = \hbar/2 \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \mathbf{L}_z = \hbar/2 \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

If we add a 4th 2-valued quantity conjugate, then re-solving to choose symmetric solutions, yields

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm j \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \pm k \end{bmatrix}$$

where i, j, k are quaternions. The corresponding operators are

$$\mathbf{K}_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{K}_1 = \begin{bmatrix} 0 & -i\sigma_x \\ i\sigma_x & 0 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 0 & -i\sigma_y \\ i\sigma_y & 0 \end{bmatrix}, \mathbf{K}_3 = \begin{bmatrix} 0 & -i\sigma_z \\ i\sigma_z & 0 \end{bmatrix}$$

which are recognizable as variants of the Dirac’s matrices.

11 The Zeroth Ticker-Tape Conjecture

The formalism of Quantum Mechanics results from the algebraic extension of classical probability theory to handle non-commuting observables.

12 The First Ticker-Tape Conjecture

This conjecture is well-known but often only stated implicitly.

The commutator relationships between measurement operators define “what we are measuring”.

The proposition has several advantages:

1. The algebraic relationships between operators can be extracted (with some caveats) experimentally from a measurement stream, even though this may be very computationally expensive.
2. Commutators do not require Classical Physics to pre-define the concepts required for use by Quantum Mechanics.
3. Commutators naturally define a “scale” and provide a natural mechanism for the introduction of constants. For example, suppose that

$$[\mathbf{X}_i, \mathbf{X}_j] = g_{ij}(\mathbf{X}_1 \dots \mathbf{X}_n)$$

If one of the devices, \mathbf{X}_i , is rescaled ($\mathbf{X}_i \rightarrow f(\mathbf{X}_i)$) then, depending on the algebra, there may be a detectable change in the commutator relations

13 Principle of Scale

A set of devices $\{\mathbf{X}_i, \dots, \mathbf{X}_n\}$ can be assigned meaning only if any rescaling (re-calibration) of the devices is detectable.

The commutator relations provide a method of ensuring different separated measuring devices use the same units.

Example 1: (Angular Momentum) Three devices are related by the commutator relationship

$$\begin{aligned}[\mathbf{L}_x, \mathbf{L}_y] &= i \hbar \mathbf{L}_z \\ [\mathbf{L}_y, \mathbf{L}_z] &= i \hbar \mathbf{L}_x \\ [\mathbf{L}_z, \mathbf{L}_x] &= i \hbar \mathbf{L}_y\end{aligned}$$

Rescale $\mathbf{L}_x \rightarrow \mathbf{L}_x' = \lambda \mathbf{L}_x$, $\lambda \neq 0$, $\lambda \neq 1$ then

$$\begin{aligned}[\mathbf{L}_x', \mathbf{L}_y] &= i \lambda \hbar \mathbf{L}_z \\ [\mathbf{L}_y, \mathbf{L}_z] &= i \hbar \left(\frac{1}{\lambda}\right) \mathbf{L}_x' \\ [\mathbf{L}_z, \mathbf{L}_x'] &= i \lambda \hbar \mathbf{L}_y\end{aligned}$$

The rescaling of \mathbf{L}_x can be partially hidden by the simultaneous rescaling \mathbf{L}_y or \mathbf{L}_z . If $\mathbf{L}_z \rightarrow \mathbf{L}_z' = \lambda \mathbf{L}_z$, then

$$\begin{aligned} [\mathbf{L}_x', \mathbf{L}_y] &= i \mathbf{h} \mathbf{L}_z' \\ [\mathbf{L}_y, \mathbf{L}_z'] &= i \mathbf{h} \mathbf{L}_x' \\ [\mathbf{L}_z', \mathbf{L}_x'] &= i \lambda^2 \mathbf{h} \mathbf{L}_y \end{aligned}$$

However no further scaling $\mathbf{L}_y \rightarrow \mathbf{L}_y' = \mu \mathbf{L}_y$, $\mu \neq 0, \mu \neq 1$ (below) can disguise the original change of scale.

$$\begin{aligned} [\mathbf{L}_x', \mathbf{L}_y'] &= i \mu \mathbf{h} \mathbf{L}_z' \\ [\mathbf{L}_y', \mathbf{L}_z'] &= i \mu \mathbf{h} \mathbf{L}_x' \\ [\mathbf{L}_z', \mathbf{L}_x'] &= i \lambda^2 \left(\frac{1}{\mu} \right) \mathbf{h} \mathbf{L}_y' \end{aligned}$$

Example 2: (Position / Momentum) Two devices are related by commutator relationship $[\mathbf{x}, \mathbf{p}] = \mathbf{h}$.

Rescale $\mathbf{x} \rightarrow \mathbf{x}' = \lambda \mathbf{x}$ (change in the choice of units), then

$$[\mathbf{x}', \mathbf{p}] = [\lambda \mathbf{x}, \mathbf{p}] = \lambda \mathbf{h}.$$

The original scaling however can be hidden by rescaling \mathbf{p} . I.e. $\mathbf{p} \rightarrow \left(\frac{1}{\lambda} \right) \mathbf{p}$.

Rescale $\mathbf{x}^* = \mathbf{x} + \lambda$ (change in the zero mark of the measuring device), then again

$$[\mathbf{x}^*, \mathbf{p}] = [\mathbf{x}, \mathbf{p}] = \mathbf{h}.$$

The choice of a different zero mark has no effect on the commutator relations. I.e. It is possible to replace \mathbf{x} with \mathbf{x}^* with no effect on the physics.

Example 3: (Position and Momentum in 3D + Angular Momentum). There are nine devices in all. The non-zero commutator relations are shown below.

$$\begin{array}{lll} [\mathbf{L}_x, \mathbf{L}_y] = i \mathbf{h} \mathbf{L}_z & [\mathbf{L}_y, \mathbf{L}_z] = i \mathbf{h} \mathbf{L}_x & [\mathbf{L}_z, \mathbf{L}_x] = i \mathbf{h} \mathbf{L}_y \\ [\mathbf{x}, \mathbf{p}_x] = \mathbf{h} & [\mathbf{y}, \mathbf{p}_y] = \mathbf{h} & [\mathbf{z}, \mathbf{p}_z] = \mathbf{h} \\ [\mathbf{L}_x, \mathbf{y}] = i \mathbf{h} \mathbf{z} & [\mathbf{L}_y, \mathbf{z}] = i \mathbf{h} \mathbf{x} & [\mathbf{L}_z, \mathbf{x}] = i \mathbf{h} \mathbf{p}_y \\ [\mathbf{L}_x, \mathbf{p}_y] = i \mathbf{h} \mathbf{p}_z & [\mathbf{L}_y, \mathbf{p}_z] = i \mathbf{h} \mathbf{p}_x & [\mathbf{L}_z, \mathbf{p}_x] = i \mathbf{h} \mathbf{p}_y \end{array}$$

We know that any rescaling of the angular momentum devices is immediately detectable. The position and momentum operators can be undetectably rescaled provided that if $\mathbf{x} \rightarrow \mathbf{x}' = \lambda \mathbf{x}$, then

$$\mathbf{p} \rightarrow \left(\frac{1}{\lambda} \right) \mathbf{p}.$$

However a single rescaling in 1 dimension (such as $y \rightarrow y' = \lambda y$) can be detected. I.e. the angular momentum commutator relationships provide a mechanism for comparing the scaling of position and momentum devices in different orientations.

It is tempting to complete the calibration process by somehow appealing to Einstein's Relativity and the constancy of the speed of light. If we have a clock, then distance can be scaled in any direction using $\delta x = c \delta t$. I.e. the constancy of the speed of light provides a mechanism for comparing the scaling of position and momentum devices at different locations. The problem is that this requires another layer of abstraction, not yet developed, to define the concept of "velocity".

14 Time

Naive observers, equipped only with Mach devices, do not have access to a clock. How would such an observer measure time?

14.1 Pauli's Theorem

Pauli's Theorem famously states that if the Hamiltonian \mathbf{H} is bounded from below, then there is no (time) operator \mathbf{T} which obeys the expected commutator relation $[\mathbf{H}, \mathbf{T}] = i \mathbf{h}$, and has unbounded eigenvalues.

Pauli's Theorem strongly suggests that there is no such thing as a time operator in Quantum Mechanics. No-go theorems always need to be treated with caution. (Ironically complex numbers and quaternions can be viewed as devices to get around mathematical no-go theorems). The best attempts at building a quantum clock so far are statistical in nature. We go with the assumption that that's as good as it gets.

14.2 Ensemble Clocks (Quantum Egg-Timers)

The diagram below shows a ensemble quantum clock frequently described in the literature *

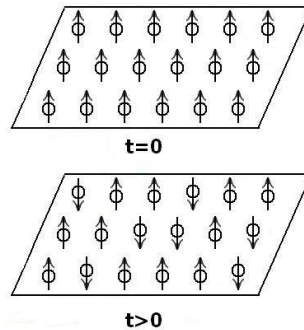


Figure 9
Quantum Clock.

The device consists of a large number of “particles” all prepared in identical states, denoted \uparrow . The “particles” spontaneously decay to a second state, denoted \downarrow . Each transition is believed, from the analysis of pass behaviour, to be statistically independent from any other.

A statistical estimate of the time past since the assembly of particles was prepared ($t = 0$) can be made by counting the number of particles that have changed state ($t > 0$). A clock can be designed with arbitrarily high confidence by increasing the number of particles in the ensemble.

If $N(t)$ = expected proportion of “particles” in the \uparrow state at time t , then

$$N(t) = e^{-\lambda t}$$

Solving for the time t :

$$t \approx -\ln(N) / \lambda$$

If the clock was originally prepared so that all the particles were initially in the state \uparrow , then at some point all or most of the particles will be in the \downarrow state. At that point, the clock operates in reverse - it measures the transitions \uparrow to \downarrow . (Flipping the Egg-Timer).

Flipping the Egg-Timer also provides us with ability to accumulate statistics and check the operation of the clock. The individual particles that make up the clock should be independent from

each other ($P(\mathbf{AB}) = P(\mathbf{A})P(\mathbf{B})$ where \mathbf{A} and \mathbf{B} represent specific particle state transitions). Each particle should be equally likely to make a transition “at any time”. I.e. If there are N particles in the ensemble, the the average transition for any particle should occur at $N/2$.

* Type “ensemble quantum clock” into Google to see this.

15 The Second Ticker-Tape Conjecture

Time is a statistical concept.

16 Feynman’s Rules

We follow the argument put forward by Ariel Caticha^[2] which builds on work done by R.T. Cox.^[3], E.T. Jaynes and others.

Definition: If $\mathbf{H}_A = (\mathbf{m}_1, \dots, \mathbf{m}_k)$ and $\mathbf{H}_B = (\mathbf{m}_{k+1}, \dots, \mathbf{m}_{k+m})$ are histories, then

$$\mathbf{H}_A \wedge \mathbf{H}_B = (\mathbf{m}_1, \dots, \mathbf{m}_k, \mathbf{m}_{k+1}, \dots, \mathbf{m}_{k+m})$$

The history \mathbf{H}_A must “follow” the history \mathbf{H}_B and not overlap in time. The operator \wedge is read as “and”.

Definition: Let \mathbf{H}_A be a history, then \mathbf{H}_B is an alternative history to \mathbf{H}_A if \mathbf{H}_B is the same as \mathbf{H}_A except that some of the measurements, other than the initial and final measurements, have different values.

Definition: If $\mathbf{H}_A = (\mathbf{m}_1, \dots, \mathbf{m}_{A_i}, \dots, \mathbf{m}_n)$ and $\mathbf{H}_B = (\mathbf{m}_1, \dots, \mathbf{m}_{B_i}, \dots, \mathbf{m}_n)$ are alternative histories which differ in the value of the i^{th} measurement, then

$$\mathbf{H}_A \vee \mathbf{H}_B = (\mathbf{m}_1, \dots, \mathbf{m}_{A_i} + \mathbf{m}_{B_i}, \dots, \mathbf{m}_n)$$

The operator \vee is read as “or”.

The operators \wedge and \vee obey the following relations:

$$\begin{aligned} a \vee b &= b \vee a \\ (a \vee b) \vee c &= a \vee (b \vee c) \\ (a \wedge b) \wedge c &= a \wedge (b \wedge c) \\ a \wedge (b \vee c) &= (a \wedge b) \vee (a \wedge c) \end{aligned}$$

If \mathbf{H} = set of possible histories for a system, the any representation $(\Psi, +, \times)$ with $\Psi: \mathbf{H} \rightarrow \Omega$ and

$$\begin{aligned} \Psi(a \vee b) &= \Psi(a) + \Psi(b) \\ \Psi(a \wedge b) &= \Psi(a) \times \Psi(b) \end{aligned}$$

would need to carry these properties across.

Caticha^[2] shows that for the general solution for complex valued representations $+$ and \times are the standard addition and multiplication operators.

17 Hamiltonian

Once we have a clock, we can relax any requirement that transition probability are stable, and that always $P(\Psi | \Psi) = 1$. The Hamiltonian can be introduced simply by “differentiating” the state vector with respect to time.

18 Comments

The Copenhagen Interpretation has now become so polluted as a brand that it is now virtually impossible to determine what it is. The Ticker-Tape Interpretation is based on Copenhagen (I would hope that Bohr, Heisenberg and Born would recognise it and approve) but rejects the view of Kant and Bohr that classical notions of space-time and even Physics are a necessary prerequisite for the formulation of Quantum Physics.

The Ticker-Tape Interpretation is exciting – it needs naturally to a research program: What devices satisfy Principle of Scale? How to we generate “space” and “particles” from our measurement streams? How can the interpretation be reconciled with other research streams such as String Theory?

19 References

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